Activity 6: Subtraction
Recorder's Report

Manager: Reader:

Recorder: Driver:

Date: Score: Satisfactory   /   Not Satisfactory

Record your team's answers to the key questions (marked with) below.

  a) Model 1, Question #2

  b) Model 2, Question #7

  c) Model 3, Question #12

  d) Model 3, Question #16

  e) Model 4, Question #21, a–c

Activity 6: Subtraction

We said earlier that most math operations can be simplified to addition. In this lesson we will build on earlier exercises dealing with binary and with digital logic gates to understand how computers do subtraction.

Content Learning Objectives

After completing this activity, students should be able to:

- Use nine's complement to subtract without borrowing; and,
- Represent negative numbers in binary using two's complement.

Process Skill Goals

During the activity, students should make progress toward:

- Reflect on how what how the team could work and learn more effectively.

Sources

- odometer link here
- clock: https://pngio.com/images/png-a1553797.html

Model 1   Subtraction without Borrowing

Questions (10 min)   **Start time:**

1. Complete the problem below. With subtraction, what complexity arises analogous to (but more difficult than) the carry step in addition?

	2	5	3
-	1	7	6

2. Given the expression $x - y$ (where y is no more than three digits), what decimal value for x will guarantee that you never have to borrow?

3. To avoid borrowing, we can use algebra to rewrite $x - y$ as follows:

$$x - y = x - y + 0$$
$$= x - y + (1000 - 1000)$$
$$= x - y + (999 + 1 - 1000)$$
$$= x + (999 - y) + 1 - 1000$$
$$= (999 - y) + x + 1 - 1000$$

Complete the problems in the table below and confirm that the result is the same.

		9	9	9
-		1	7	6
+		2	5	3
+				1
-	1	0	0	0

The result of subtracting a number from nines gives us a nines' complement. Note that while the word *compliment* means to say something nice, the word *complement* means "a thing that completes." Adding a number and its nines' complement gives us all (or just) nines.

4. Of course, in a computer with digital logic we will be using binary instead of decimal. Given the expression $x - y$ (where y is no more than 8 digits), what binary value for x will guarantee that you never have to borrow?

5. Complete the binary subtraction problem below.

	1	1
-	1	0

a) What pattern do you notice when you compare the bits in rows 2 and 3 column by column (the subtrahend and the difference)?

b) With respect to each column, which truth table takes one input (from line 2) and gives one output (line 3)?

Model 2   Wrap-Around

Questions   (10 min)   **Start time:**

6. Consider an odometer with only five digits (left of the decimal).

a) If it reads 99999 and you add one mile, what will it show?

b) If you have a limited number of digits, is there a way to avoid this wrap-around?

7. Consider a binary value with eight bits.

a) How many unique values could you represent (the count, not a formula)?

b) If you chose to represent only non-negative integers, what would be the range of unsigned integers (converted to decimal)?

c) If you added one to the maximum value, what would you get?

d) If you subtracted one from the minimum value, what would you get?

e) If you used one bit for a sign (indicating positive or negative), how many bits would be left for the number?

f) How many values could be represented using the remaining bits?

g) What is the range of unsigned integers you could represent in the remaining bits?

h) What would be the range of signed integers you could represent with a single sign bit and seven value bits?

i) How many unique values does that represent?

j) Are there any missing values? If so, where did they go?

Next, we will consider another approach to representing negative numbers that allows us to store the maximum number of values and allows adding negative numbers without any special work.

Model 3   Negative Numbers

Questions (15 min)   **Start time:**

8. Consider a traditional analog clock face.

a) How many unique hours can it represent?

b) What is the range of hours it represents?

c) If we think of 3:00 as "three hours after noon" (or "+3"), what hour position on the clock is represented by "three hours before noon" ("-3")?

d) What range of integers would support the same number of negative clock-face "hours" as nonnegative clock-face "hours"?

9. Complete the following table to map the typical clock face hours (line 1) to the hours before and after noon (line 2). (Hint: start at each end and work to the middle.)

12	1	2	3	4	5	6	7	8	9	10	11

10. With this relabeled clock, answer the following questions:

a) If we start at 2 and add 2, what do we get?

b) If we start at 2 and subtract 1, what do we get?

c) If we start at 2 and subtract 2, what do we get? (Hint: not 12)

d) If we start at 2 and subtract 5, what do we get?

e) If we start at -2 and subtract 2, what do we get?

f) If we start at -2 and add 6, what do we get?

g) So far this maps very nicely to "normal" arithmetic. But if we start at 5 and add 3, what do we get? (Hint: not 8!)

h) If we start at -5 and subtract 4, what do we get? (Hint: not -9!)

11.   Using a similar approach, fill in the following table to map selected unsigned values [0, 255] to signed values [-128, 127]. This is called a two's complement.

0	1	2	...	126	127	128	129	130	...	254	255
			...						...		

12. Convert some of these binary values to unsigned and signed decimal:

Binary	Unsigned Decimal	Signed Decimal
0000 0000		
0000 0001		
0111 1110		
0111 1111		
1000 0000		
1000 0001		
1111 1110		
1111 1111		

13. How does the high-order (left-most) bit correlate with the sign?

14. Perform some addition of signed binary numbers and note how the sign works:

11	0000 1011	-118	1000 1010
+ 5	+ 0000 0101	+ 44	+ 0010 1100
106	0110 1010	-22	1110 1010
+ -63	+ 1100 0001	+ -86	+1010 1010

This suggests that another way to do subtraction is just to add the negative value. So how do we get a negative? Compare a few examples and note that in each case, if you added the values you would get zeros in the right-most (low order) 8 bits (which is, of course, correct).

$$1 = 0000\ 0001 \qquad -1 = 1111\ 1111$$
$$2 = 0000\ 0010 \qquad -2 = 1111\ 1110$$
$$126 = 0111\ 1110 \qquad -126 = 1000\ 0010$$
$$127 = 0111\ 1111 \qquad -127 = 1000\ 0001$$

15. For each positive value above (1, 2, 126, and 127), write out its one's complement or inverse (replace 0 with 1, 1 with 0).

16. How does the inverse compare with the negative value? What would you add to the inverse to get the negative? Learn this useful rule for computing the negative of a number!

Model 4   Encoding in Binary

While we said that there isn't anything special about decimal (base ten), there is something special about binary: it is the simplest number system possible and forms the foundation of all digital computers. Anything that can be mapped to discrete values can be encoded in binary; the only requirement is that everyone who uses the code must understand the mapping.

Refer to Model 4 above as your team develops consensus answers to the questions below.

Questions (15 min)   Start time:

17. "Hang a lantern aloft in the belfry arch / Of the North Church tower as a special light, — / One, if by land, and two. if by sea ... " Map Paul Revere's code to binary.

Code	Meaning
00	
01	
10	
11	"by sea"

18. The WWU CS department has four professors, Preston Carman, James Foster, John Foster, and Natalie Smith-Gray. Assign each a unique ID using just two bits.

Code	Professor
	Preston Carman
	James Foster
	John Foster
	Natalie Smith-Gray

19. How many bits would be required to encode the following:

   a) The letter grades A, B, C, D, F? (Hint: not 5!)

b) The 30 students enrolled in CPTR 280?

c) All 34 students if we admitted everyone on the wait list?

20. If you were using four bits to represent a non-negative integer (to keep things simple), what is the range of numbers you could encode (that is, what would they convert to in decimal)?