

Activity 5: Addition

Recorder's Report

Manager:

Reader:

Recorder:

Driver:

Date:

Score: Satisfactory / Not Satisfactory

Record your team's answers to the key questions (marked with ) below.

a) Model 1, Question #3

b) Model 3, Question #8.g

c) Model 4, Question #9

Activity 5: Addition

Just as number systems can be simplified to binary, most math operations can be simplified to addition. In this lesson we will build on earlier exercises dealing with binary and with digital logic gates to understand how computers do addition and subtraction.

Content Learning Objectives

After completing this activity, students should be able to:

- Describe the logic circuits used in adding binary numbers;
- Use nine's complement to subtract without borrowing; and,
- Represent negative numbers in binary using two's complement.

Process Skill Goals

During the activity, students should make progress toward:

- Reflect on how teams can ensure all members participate.



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Model 1 Addition with Carry

Questions (10 min)

Start time:

1. Complete the decimal addition problem in the table below, then answer the following questions:

	2	6
+	4	5

- What is the sum of $6 + 5$ (answer after completing the table)?
- How many digits are in that value?
- Where did you put the least significant (right-most) digit?
- Where did you put the most significant (left-most) digit?
- When you have a one that “carries” to the next column, how is it used?
- For the expression $(9 + 8 + 7)$, do you add all three numbers at once or do you first add two of them and then add the third to the intermediate sum?
- When adding two N -digit numbers, what is the maximum number of digits for the sum?

2. The binary addition table (with four totals) is much simpler than the decimal addition table (with 100 totals).

- Complete the left table below (labeled "total") with a two-digit sum in each of the empty cells.
- Complete the "sum" table with the least-significant digit (right-most) from the first table.
- Complete the "carry" table with the most-significant digit (left-most) from the first table.

total	$A = 0$	$A = 1$
$B = 0$		
$B = 1$		

sum	$A = 0$	$A = 1$
$B = 0$		
$B = 1$		

carry	$A = 0$	$A = 1$
$B = 0$		
$B = 1$		

3. The truth tables below were developed in an earlier exercise.



- Which one matches the sum table?
- Which one matches the carry table?

AND	0	1	OR	0	1	NOT		NAND	0	1	NOR	0	1	XOR	0	1
$B = 0$	0	0	$B = 0$	0	1	$B = 0$	1	$B = 0$	1	1	$B = 0$	1	0	$B = 0$	0	1
$B = 1$	0	1	$B = 1$	1	1	$B = 1$	0	$B = 1$	1	0	$B = 1$	0	0	$B = 1$	1	0

Model 2 Digital Circuit for a Half Adder

Questions (10 min)

Start time:

4. The following symbols are used to represent digital logic circuits that produce the truth tables described above. Based on your answer to question 3, label one of them sum and another one of them carry.

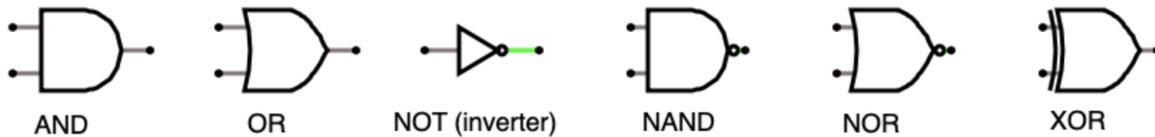


Figure 1: Logic Gate Symbols

5. Complete Figure 2 by identifying the output value for each of the gates given the input values for A and B:

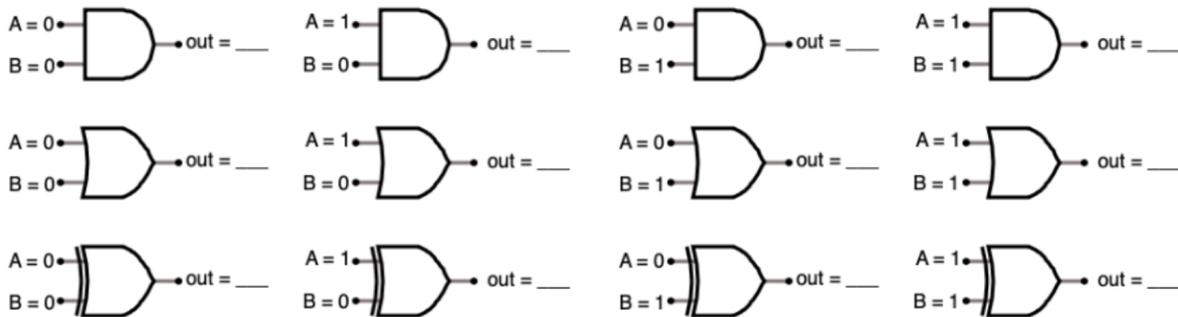


Figure 2: Logic Gate Outputs

6. Follow the instructions in (a) and (b) below to complete the circuit diagram in Figure 3 so that it can add two binary digits A and B:

- Connect A and B to the appropriate input lines on each of the two gates. (Hint: see question 5.)
- Label the outputs sum and carry. (Hint: see question 4.)

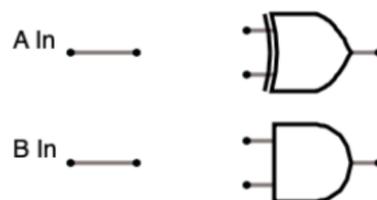


Figure 3: Adding Two Binary Digits

A circuit that has two inputs and a sum and carry output is called a Half Adder and has a special symbol (Figure 4).

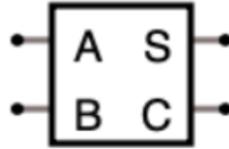


Figure 4: Half Adder

Model 3 Digital Circuit for a Full Adder

Questions (15 min)

Start time:

7. Add the binary numbers $110 + 011$ in the table to the right (fill each of the non-shaded cells). (Hint: see question 2.)

	digit	3	2	1	0
	carry				
	A		1	1	0
+	B		0	1	1
	sum				

- What is the value for carry for digit 2 in this problem (the third column from the right)? Will it always be that even for other values of A & B?
- What is the value for carry for digit 1 in this problem? Will it always be that even for other values of A & B?
- What is the value for carry for digit 0 in this problem? Will it always be that even for other values of A & B?
- If we wanted the circuit for each column to be the same (simplicity/consistency), how many inputs (digits or bits) should we allow for each column?
- How many outputs (digits or bits) should we allow for each column?

f) When adding digits, does the order matter?

8. Follow the instructions below to create a Full Adder in Figure 5.

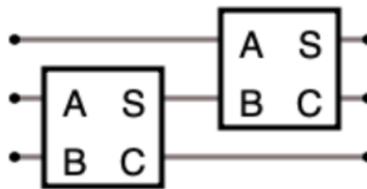


Figure 5: Adding Three Binary Digits

- Label the three inputs (on the left of the figure) for carry, A, and B. (Hint: see question 7f.)
- How many outputs should we have? (Hint: see question 7e.)
- What should the outputs be named?
- When adding two, one-bit values, will sum and carry ever both be one? (Hint: see question 2.)
- If $C = 1$ on the left Half Adder, what is the value of S on the left one?
- If $S = 0$ on the left Half Adder, what is the value of C on the right one?
- Will both C values from the Half Adders in Figure 5 ever both be one? 
- Add the appropriate gate from Figure 1 on the right of Figure 5 to complete the circuit.
- Label the outputs. (Hint: try various inputs and see what outputs you get.)

Model 4 Multi-Bit Adder

A circuit like the one built in question 8, that has three inputs and two outputs, a sum and carry, is called a Full Adder. It can be represented with either of the images in Figure 6. The difference between them is whether their inputs and outputs are on the left or right. While we have had inputs on the left so far, we will soon see how it may be convenient to have the inputs on the right.

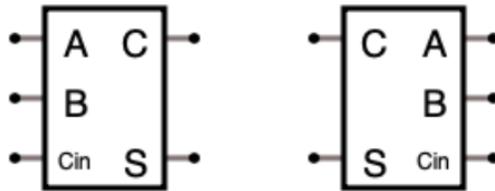


Figure 6: Full Adders

Refer to Model 4 above as your team develops consensus answers to the questions below.

Questions (15 min)

Start time:

9. To add a pair of N-digit binary numbers, how many Full Adders do we need? (Hint:  how many columns are we adding in question 7?)

10. Follow the instructions below to make a 3-Bit Adder in Figure 7 to add $A + B$.

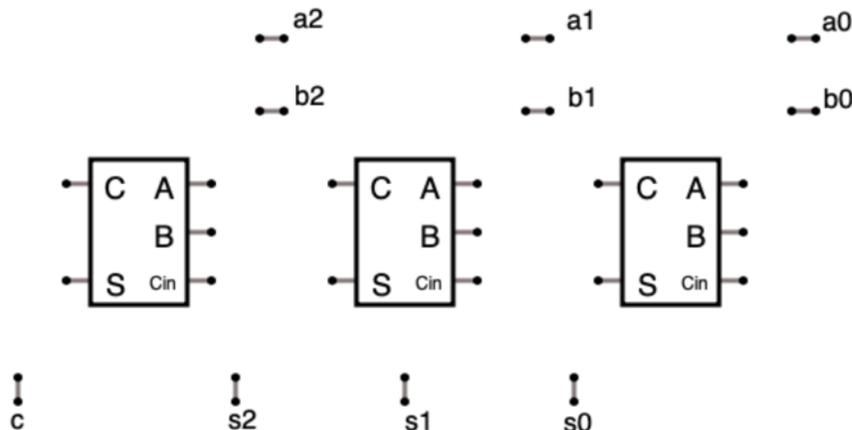


Figure 7: 3-Bit Adder

- a) Given that A is a three-bit number with the least-significant bit on the right (a_0) and the most significant bit on the left (a_2), draw lines connecting each digit of A to the appropriate Full Adder.
- b) Draw lines connecting each digit of B to the appropriate Full Adder.
- c) Draw lines connecting the S output of each Full Adder to the appropriate output (s_0 - s_2).
- d) Draw lines connecting the C output of each Full Adder to the appropriate place.
- e) Draw a component for the input for C_{in} on the least-significant Full Adder (at the right).
(Hint: what is the value needed and what circuit element provides that value?)