Activity 3: Number Systems

Recorder's Report

Manager:

Reader:

Recorder:

Driver:

Date:

Score: Satisfactory / Not Satisfactory

Record your team's answers to the key questions (marked with) below.

a) Model 1, Question #6

b) Model 2, Question #14

c) Model 3, Question #23

Activity 3: Number Systems

At the fundamental level digital computers work with high and low voltage, interpreted as ones and zeros and represented as binary numbers. Understanding how data is represented as binary numbers is important to understanding how computers work and how to writing programs.

Content Learning Objectives

After completing this activity, students should be able to:

- explain the role of exponents in positional notation; and,
- encode data in binary.

Process Skill Goals

During the activity, students should make progress toward:

- convert between numbers in different bases.

Sources

- Based in part on This activity
- Based in part on This activity

Model 1   Representing Numbers

Whether we write cat, gato, or chat, we are referring to a . Different words can be code for the same meaning. In a similar way, there are various ways of recording a number of ducks:

   VII   

Refer to Model 1 above as your team develops consensus answers to the questions below.

Questions (10 min)   Start time:

1. Write the number of ducks as a numeral and as a word.

2. Recall that in Roman Numerals, X represents the number of digits (fingers and thumbs on two hands) for a typical person. Provide the representation for the number of items in a dozen using Roman Numerals and at least two other representations.

3. While Roman Numerals has some disadvantages, it works pretty well for some addition and subtraction problems. Calculate the following and record the answer in Roman Numerals. How does the answer compare to the problem?

XI + VII =

4. Can you think of another way Roman Numerals might be easy to use, especially for someone carving dates on stone?

Our number system is known as the Hindu-Arabic or Indo-Arabic, or just Arabic number system and was brought to Europe from India by Arab mathematicians.

5. Compare the numbers II (Roman) and 11 (Arabic).

a) What is the value/meaning of the second (right-hand) of each of the numerals?

b) What is the value/meaning of the first (left-hand) of the two Roman Numerals?

c) What is the value/meaning of the first (left-hand) of the two Arabic Numerals?

d) How would you describe the difference?

6. Compare the numbers M (Roman) and 1000 (Arabic). What is the role of zero?

7. Why do Arabic Numerals not have special symbols for ten, one hundred, and one thousand?

Model 2 Positional Notation

The structure of the Hindu-Arabic number system is revealed in the way we pronounce numbers. The year one thousand, nine hundred, fifty-eight is represented as follows:

Decimal Numbers	ten-thousands	thousands	hundreds	tens	ones	one-tenths
Digit	0	1	9	5	8	0
Place (full)	10,000	1,000	100	10	1	0.1
Place (exponent notation)	10^4	10^3	10^2	10^1	10^0	10^{-1}
Digit * Place	0	1,000	900	50	8	0

Refer to Model 2 above as your team develops consensus answers to the questions below.

Questions (10 min) Start time:

8. What is the value of the exponent for the hundreds place?

9. Without changing the number represented, complete the column to the left of the thousands. What would be the digit for each additional column to the left for the given number?

10. Without changing the number represented, complete the column to the right of the ones. What would be the digit for each additional column to the right for the given number?

11. The use of ten as a base is presumably a consequence of most people having ten digits (fingers and thumbs on two hands) that are easy to use for counting. But there is nothing magical about base ten—other bases work just the same (and have some advantages, as we will see). Complete the following table to convert the octal number 1776_{eight} to base ten (use of a calculator is fine).

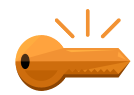
Octal Numbers	five hundred twelves	sixty-fours	eights	ones	**Total**
Digit	1	7	7	6	
Place (full)	512	64	8	1	
Place (exponent notation)	8^3	8^2	8^1	8^0	
Digit * Place					

12. Complete the following table to convert the binary number 101010_{two} to decimal.

Binary Numbers	32s	sixteens	eights	fours	twos	ones	**Total**
Digit	1	0	1	0	1	0	
Place (full)	32	16	8	4	2	1	
Place (exponent notation)	2^5	2^4	2^3	2^2	2^1	2^0	
Digit * Place							

13. Convert the number 10010000_{two} to decimal.

14. What two symbols (digits) are used to represent quantities in a binary system?



15. In binary, what do all odd numbers have in common?

Model 3 Convert Between Bases

There are (at least) two approaches to converting a decimal number to another base. Each approach involves divisions (by powers of the base) and special treatment of the (integer) quotient and remainder.

One approach to converting a decimal number to another base is to start building the number from the left (with the most significant digit) and working to the right (with the least significant digit). Observe how we use the following table to convert 1000_{ten} to 1750_{eight}.

Step	1	2	3	4
Start and Remainder	1,000	488	40	0
Place	8^3	8^2	8^1	8^0
Divide by Place	$\div$ 512	$\div$ 64	$\div$ 8	$\div$ 1
Quotient	1	7	5	0

Refer to Model 3 above as your team develops consensus answers to the questions below.

Questions (15 min) Start time:

16. Complete the following table to convert $10{,}000_{ten}$ to octal.

Start and Remainder					
Place	8^4	8^3	8^2	8^1	8^0
Divide by Place	$\div$ 4096	$\div$ 512	$\div$ 64	$\div$ 8	$\div$ 1
Quotient					

17. Complete the following table to convert 100_{ten} to binary.

Start and Remainder							
Place	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Divide by Place	$\div$ 64	$\div$ 32	$\div$ 16	$\div$ 8	$\div$ 4	$\div$ 2	$\div$ 1
Quotient							

18. Recall that one bit can have two values (0 and 1), and that two bits can have four values (00, 01, 10, and 11).

a) How many values can 3 bits represent? (hint: not 6!)

b) How many values can 4 bits represent?

c) How many values can 8 bits represent?

d) How many values can 10 bits represent?

e) What is the formula for how many values N bits can represent?

These values are very useful; remember them!

Another approach to converting from decimal to a base is to start building the number from the right and work left. In this case we always divide by the base. Observe how we use the following table to convert 1000_{ten} to 1750_{eight}.

Step	4	3	2	1
Start and Quotient	1	15	125	1000
Divide by Base	$\div$ 8	$\div$ 8	$\div$ 8	$\div$ 8
Remainder	1	7	5	0

19. Complete the following table to convert $10{,}000_{ten}$ to octal.

Start and Quotient					
Divide by Base	$\div$ 8	$\div$ 8	$\div$ 8	$\div$ 8	$\div$ 8
Remainder					

20. Complete the following table to convert 100_{ten} to binary.

Start and Quotient							
Divide by Base	$\div$ 2	$\div$ 2	$\div$ 2	$\div$ 2	$\div$ 2	$\div$ 2	$\div$ 2
Remainder							

As a general rule, converting between non-decimal bases is done by converting to decimal and then the target base (unless you want to learn multiplication and division in other bases!). But shortcuts are possible when converting between bases where one is a power of the other. Specifically, converting between binary and octal (and later hexadecimal) is trivial. Note that each single octal digit (0-7) is always represented by exactly three binary digits, so you can build (or memorize) a simple conversion table.

21. Complete the following octal to binary conversion table:

Octal	7	6	5	4	3	2	1	0
Binary								

22. What is 4321_{eight} in binary?

23. What is 11010101_{binary} in octal?

Model 4 Hexadecimal

So far, we have looked at number systems with bases of ten or less. But if we had six fingers per hand, then we would likely grow up using base twelve (which is evenly divisible by 2, 3, 4, and 6, so has advantages!).
As it happens, while octal was popular in the early days of computers, it is more common today to use hexadecimal (base 16) to represent internal values. By convention, we use A-F to represent ten through fifteen.

Refer to Model 4 above as your team develops consensus answers to the questions below.

Questions (15 min) Start time:

24. Complete the following table to convert the hexadecimal number $7E3_{sixteen}$ to base ten.

Hexadecimal Numbers	two hundred fifty-sixes	sixteens	ones	**Total**
Digit	7	E	3	
Place (full)	256	16	1	
Place (exponent notation)	16^2	16^1	16^0	
Digit * Place				

25. Complete the following table to convert $99{,}324_{ten}$ to hexadecimal.

Start and Quotient					
Divide by Base	$\div$ 16	$\div$ 16	$\div$ 16	$\div$ 16	$\div$ 16
Remainder					

26. As an extension to question 21, complete the following hexadecimal to binary table:

Hexadecimal	F	E	D	C	B	A	9	8
Binary								

27. What is $1101'0101_{binary}$ in hexadecimal (the apostrophe character is used to group four binary digits, much like commas are used to group three decimal digits)?

28. Instead of using a subscript with the base spelled out as a word, many languages use a prefix of “0x” to designate a hexadecimal number. What is 0x7C1 in binary?